# ΜΕΜ-205 Περιγραφική Στατιστική

**Τμήμα Μαθηματικών και Εφ. Μαθηματικών, Πανεπιστήμιο Κρήτης**

Κώστας Σμαραγδάκης (kesmarag@gmail.com)

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# Φυλλάδιο 3/3

$$Y_t^* = \sum_{u=-1}^{1} a_u Y_{t-u}$$

$Y_t = \{85, 113, 130, 111, 140, 164, 143, 170, 203, 188, 206, 227\}$ $\quad p=3$

$\alpha = [1/3, 1/3, 1/3]^T$ $\quad 2s+1 = 3 \Leftrightarrow s=1$

$Y_t^* = \{109.3, 118, 127, 128.3, 149, 159, 173.3, 183.3, 200.3, 207\}$

$Y_1^*$ $Y_3^*$ $Y_{11}^*$

$D_t = Y_t - Y_t^*, \quad t = 2, \ldots, 11$ $\quad D_t \sim S_t + R_t$

$D_t = \{-24.3, 18, -16, 1.7, 15, -16, -23.3, 18.7, -12.3, -1\}$ $\sim S_t + R_t$

$S_1 = S_4 = S_7$

$S_2 = S_5 = \ldots$

$S_2+R_2$ $\quad S_3+R_3$ $\quad S_1$ $\quad S_2$ $\quad S_3$ $\quad S_1$ $\quad S_2$ $\quad S_3$ $\quad S_1$ $\quad S_{11}+R_{11}$ $\quad S_2$

$\bar{D}_1 = \frac{1}{3}(-16 - 16 - 12.3) = -14.77$

$\bar{D}_2 = \frac{1}{4}(3.7 + 1.7 - 3.3 - 1) = 0.275$

$\bar{D}_3 = \frac{1}{3}(12 + 15 + 18.7) = 15.23$

$\frac{1}{3}\sum_{j=1}^{3} \bar{D}_j = 0.245$

$\hat{S}_1 = \bar{D}_1 - 0.245 = -15.015$

$\hat{S}_2 = \bar{D}_2 - 0.245 = 0.03$

$\hat{S}_3 = \bar{D}_3 - 0.245 = 14.985$

$\{-15.015, 0.03, 14.985, -15.015, 0.03, \ldots\}$

$Y_t - \hat{S}_t = \{85 + 15.015, 113 - 0.03, \ldots\}$

$T_t + R_t$

$y_t = \{85, 113, 130, 111, 140, 164, 143, 170, 207, 188, 206, 227\}$

#1 $y_{n-1} \to y_n$

$\{(85,113),(113,130),(130,111),\dots,(206,227)\}$ $\quad \bar{y}_{n-1} = 150.63$, $\quad \bar{y}_n = 163.54$

$b = \frac{SS_{y_{n-1},y_n}}{SS_{y_{n-1},y_{n-1}}} = 0.84$ $\quad a = \bar{y}_n - b\,\bar{y}_{n-1} = 37.0108$ $\quad \hat{y}_n = 37.01 + 0.84\, y_{n-1}$, $n \geq 2$

$e_n^{(1)} = y_n - (37.01 + 0.84\, y_{n-1})$ $\quad n \geq 2$ $\quad e_n^{(1)} = [-4.59, 1.93, 35.21, \dots, -16.95]$

($n=2$, $n=3$)

#2

$y_{n-1} \to y_{n-2}$

$\bar{y}_{n-2} = 150.63$ $\quad \bar{y}_{n-1} = 163.54$

$\{(113,85),(130,113),\dots,(227,206)\}$

$b = \frac{SS_{y_{n-1},y_{n-2}}}{SS_{y_{n-1},y_{n-1}}} = 0.86$ $\quad a = \bar{y}_{n-2} - b\,\bar{y}_{n-1} = 9.99$

$\hat{y}_{n-2} = 9.99 + 0.86 \cdot y_{n-1}$

$e^{(2)}_{n-2} = y_{n-2} - \hat{y}_{n-2} = \{\overbrace{22.17}^{n=1}, 8.79, -24.55, \ldots -0.85, \overbrace{-0.79}^{n=11}\}$

$e^{(1)}_n = A + B e^{(2)}_{n-2} + \varepsilon_n, \quad n > 2$

$n = 3 \qquad e^{(1)}_3 = A + B e^{(2)}_1 + \varepsilon_1$

$\{(\overbrace{22.17}^{x}, \overbrace{1.93}^{y}), (8.79, 35.21), \ldots, (-0.85, -16.95)\}$

$n = 12$

$$PACF(a) = \frac{SS_{xy}}{\sqrt{SS_{xx}\, SS_{yy}}}$$

$\sigma^2_{\hat{x}_{n,n}} = (1 - k_n)^2 \sigma^2_{\hat{x}_{n,n-1}} + \sigma^2_{\hat{x}_{n-1,n-1}} + \sigma_z^2$

$k_n = \frac{\sigma^2_{\hat{x}_{n,n-1}}}{\sigma^2_{\hat{x}_{n,n-1}} + \sigma_z^2} = \frac{\sigma^2_{\hat{x}_{n-1,n-1}} + \sigma_z^2}{\sigma^2_{x_{n-1,n-1}} + \sigma_1^2 + \sigma_z^2}$

$y_1$ $y_2$ $y_3$ $y_4$

$y_t = \{85, 113, 130, 111, 140, 164, 143, 170, 207, 188, 206, 227\}$

$y_t = A + B^{(1)} y_{t-1} + B^{(2)} y_{t-2} + B^{(3)} y_{t-3} + \varepsilon_t \quad t \geq 4$

$\{(130, 113, 85, 111), (111, 130, 113, 140), \ldots, (206, 188, 207, 227)\}$

$$X = \begin{bmatrix} 1 & 130 & 113 & 85 \\ 1 & 111 & 130 & 113 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 206 & 148 & 207 \end{bmatrix}_{9 \times 4} \qquad \beta = \begin{bmatrix} \hat{\alpha} \\ \beta^{(1)} \\ \beta^{(2)} \\ \beta^{(3)} \end{bmatrix}_{4 \times 1} \qquad y = \begin{bmatrix} 111 \\ 140 \\ \vdots \\ 227 \end{bmatrix}_{9 \times 1}$$

`np.linalg.inv(np.dot(X.T, X))`

$$\beta = (X^T X)^{-1} X^T y$$

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