MEM-205 Περιγραφική Στατιστική

Τμήμα Μαθηματικών και Εφ. Μαθηματικών, Πανεπιστήμιο Κρήτης

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Άσκηση 1

$\{(10,18),(20,16),(30,36),(40,30)\}$

$Y_n = A + BX_n + \varepsilon_n$

$b = \frac{SS_{xy}}{SS_{xx}}$

$$SS_{xy} = \sum_{n=1}^{N}(x_n - \bar{x})(y_n - \bar{y}) = \sum_{n=1}^{N} x_n y_n - \frac{\sum x_n \sum y_n}{N} = 280$$

$$SS_{xx} = \sum_{n=1}^{N}(x_n - \bar{x})^2 = \sum_{n=1}^{N} x_n^2 - \frac{(\sum x_n)^2}{N} = 500$$

$$b = \frac{280}{500} = 0.56$$

$$a = \bar{y} - b\bar{x} = \frac{\sum y_n}{4} - b\frac{\sum x_n}{4} = 11$$

$$\hat{y}_n = a + bx_n$$

	x	y	xy	x^2
	10	18	180	100
	20	16	320	400
	30	36	1080	900
	40	30	1200	1600
+	100	100	2780	3000

Άσκηση 2

$\{(x_1, y_1), \ldots, (x_N, y_N)\} \longrightarrow \{(x_1, y_1+\alpha), \ldots, (x_N, y_N+\alpha)\}$

$r_1 = \dfrac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}} \xrightarrow{y_n' = y_n + \alpha}$

$SS_{xy'} = ;$

$SS_{y'y'} = ;$

$SS_{xy'} = \sum (x_n - \bar{x})(y_n' - \bar{Y}') =$

$= \sum (x_n - \bar{x})(y_n \cancel{+\alpha} - \bar{Y} \cancel{-\alpha}) = SS_{xy}$

$\bar{Y}' = \frac{1}{N} \sum_{n=1}^{N} y_n' = \frac{1}{N} \sum_{n=1}^{N} (y_n + \alpha) =$

$= \frac{1}{N} \sum y_n + \alpha = \bar{Y} + \alpha$

$SS_{y'y'} = \sum (y_n + \alpha - \bar{Y} - \alpha)^2 = SS_{yy}$

$r_1 = r_2$

Άσκηση 4

$\{(j,\ 4j+10(-1)^j)\}_{j=1}^{101}$ $\qquad \hat{\mu}_{y|x^*} \qquad \hat{y}^*$ $\qquad b=\dfrac{SS_{xy}}{SS_{xx}}$ $\qquad \displaystyle\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$

$$\bar{x} = \frac{\sum j}{101} = \frac{\frac{101\cdot 102}{2}}{101} = 51$$

$$\bar{y} = \frac{4\sum j + 10\sum^{101}(-1)^j}{101} = 4\cdot 51 - \frac{10}{101} = 204 - \frac{10}{101}$$

$(\cancel{-1+1-1+1}+\cdots-1)$

$$\underbrace{(-1+2)+(-3+4)+\cdots+(-99+100)}_{50\cdot 1} - 101 = -51$$

$$SS_{xy} = \sum x_n y_n - \frac{\sum x_n \sum y_n}{N}$$

$$\sum x_n y_n = \sum n(4n+10(-1)^n) = 4\sum n^2 + 10\sum_{n=1}^{101} n(-1)^n = 4\cdot 101\cdot 102\cdot 203/6 - 510$$

$$SS_{xx} = \sum n^2 - \frac{(\sum n)^2}{101} \qquad b \approx 4$$

$$SS_{yy} = \sum y_n^2 - \frac{(\sum y_n)^2}{101}$$

$$\sum y_n^2 = \sum(4n+10(-1)^n)^2 = \sum(16n^2 + 80n(-1)^n + 100)$$

$$= 16\sum n^2 + \underline{80}\sum n(-1)^n + 100\cdot 101$$

$$SSE = SS_{yy} - b\, SS_{xy}$$

$$S_e = \sqrt{\frac{SSE}{N-2}}$$

$$x^* = 8$$

$$\hat{\mu}_{y|x^*} = \bar{y} + b(x^* - \bar{x}) = 31.9$$

$$\left[\hat{\mu}_{y|x^*} - \underset{1.98}{t} S_e\sqrt{\frac{1}{N}+\frac{(x^*-\bar{x})^2}{SS_{xx}}},\ \hat{\mu}_{y|x^*} + \underset{1.98}{t} S_e\sqrt{\frac{1}{N}+\frac{(x^*-\bar{x})^2}{SS_{xx}}}\right]$$

$$\left[\hat{y}^* - t\,S_e\sqrt{1+\frac{1}{N}+\frac{(x^*-\bar{x})^2}{SS_{xx}}},\ \ldots\right]$$

Άσκηση 4

Άσκηση 5

(α)

$N = 50 \qquad \underline{\hat{y} = -6 + 3x}, \quad S_e = 8, \quad SS_{xx} = 40, \quad \underline{\underline{\bar{x} = 6}}$

$x^* = 14$

$$S_{\hat{y}^*} = \underline{S_e} \sqrt{1 + \frac{1}{N} + \frac{(x^* - \bar{x})^2}{SS_{xx}}} = \qquad \hat{y}^* = -6 + 3x^* = 36$$

$$= 9.712 \qquad \alpha = 0.05$$

$$t = 2.011$$

$$df = N - 2 = 48 \qquad t \text{ row} \quad P(T < t) = 1 - \frac{\alpha}{2} = 0.975$$

$$[y^* - t S_{\hat{y}^*},\ y^* + t S_{\hat{y}^*}]$$

$$SS'_{xx} = \sum_{n=1}^{51} (x_n - \bar{x})^2 = \sum_{n=1}^{50} (x_n - \bar{x})^2 + \cancel{(x_{51} - \bar{x})}^{0\,2}$$

$$b' = \frac{SS'_{xy}}{SS'_{xx}} = b \qquad = SS_{xx}$$

(b)

$$N = 50 \xrightarrow{(6,12)} N' = 51 \qquad \bar{x}' = 6$$

$$SS'_{xy} = SS_{xy}$$

$$\bar{y} = -6 + 3\bar{x} = -6 + 3 \cdot 6 = 12 \qquad \bar{y}' = 12$$

$$\alpha' = \bar{y}' - b'\bar{x}' = \alpha$$

$$S'_e = \sqrt{\frac{SS'_{yy} - b' SS'_{xy}}{N' - 2}} = \sqrt{\frac{N-2}{N'-2}} \sqrt{\frac{SS_{yy} - b SS_{xy}}{N-2}} = \sqrt{\frac{48}{49}}\, S_e$$

$$S'_{\hat{y}^*} = \sqrt{\frac{48}{49}}\, S_e \sqrt{1 + \frac{1}{51} + \frac{(x^* - \bar{x})^2}{SS_{xx}}} \qquad t' = 2.0096 \qquad [y^* - t' S'_{\hat{y}^*},\ y^* + t' S'_{\hat{y}^*}]$$

Άσκηση 6

$\{(40, 16.8), (50, 24.6), (60, 36.6), (70, 48)\}$

$\left(\frac{x}{10}\right)^2 \approx y \qquad x \approx 10\sqrt{y} \qquad g(y) = 10\sqrt{y}$

$\{(40, 40.99), (50, 49.6), (60, 60.5), (70, 69.28)\}$

$b \approx 1$

Εναλλακτικά

$g(y) = \sqrt{y}$

$b \approx 1/10$

Άσκηση 6

$$X = \begin{array}{c} \begin{array}{cccc} & K\pi & B & d^{(1)} \quad d^{(2)} \end{array} \\ \begin{bmatrix} 1 & & 1 & 0 \\ 1 & & 1 & 0 \\ 1 & & 0 & 1 \\ 1 & & 0 & \\ 1 & & 1 & \\ 1 & & 0 & \end{bmatrix} \end{array} \qquad y = \begin{bmatrix} 2 \\ 6 \\ 0 \\ 5 \\ \alpha \end{bmatrix} \qquad p = (X^T X)^{-1} X^T y$$

	$d^{(1)}$	$d^{(2)}$	$d^{(3)}$
k	1	0	0
π	0	1	0
θ	0	0	1