MEM-205 Περιγραφική Στατιστική

Τμήμα Μαθηματικών και Εφ. Μαθηματικών, Πανεπιστήμιο Κρήτης

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Προσαρμογή της Εποχικότητας

$Y_t = T_t + S_t + R_t$

- Έστω S_t είναι p-periodic

$$S_t = S_{t+p}, \quad t = 1, \dots, N - p$$

$[S_1, S_2, \dots, S_N]$

- Εάν εφαρμόσουμε τον απλό κινητό μέσο p τάξης

$$S_t^* = S$$

$p = 2 \qquad 2s' = 2 \Leftrightarrow s' = 1$

$2s'+1 \qquad [\frac{1}{4s'}, \frac{1}{2s'}, \dots, \frac{1}{2s'}, \frac{1}{4s'}]^T$

$[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}]^T$

- Υποθέτουμε ότι $S_t^* = 0$, ενσωματώνοντας το S στη μακροχρόνια τάση

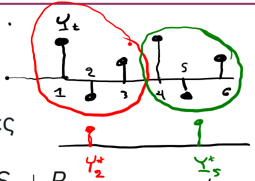
$$T_t' = T_t + S$$

$Y_t = \overbrace{T_t + S'}^{T_t} + \overbrace{S_t - S'}^{S_t} + R_t$

- Για ευκολία από εδώ και πέρα θα ενοούμε ως T_t το T_t'

$Y_t = T_t + S_t + R_t \Rightarrow Y_t - S_t = T_t + R_t$

$Y_t^* = T_t^* + S_t^* + R_t^* = T_t^* + R_t^* \sim T_t + 0 = T_t$



$p = 3$

$2s+1 = 3 \Leftrightarrow s' = 1$

$t = 1+s', \ldots, N-s'$

- Ορίζουμε τη χρονολογική σειρά με τις διαφορές

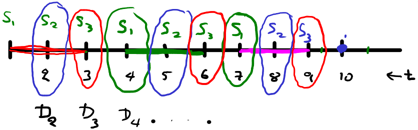
$$D_t = Y_t - Y_t^* \sim S_t + R_t \quad , t = 1+s', \ldots, N-s'$$

$T_t + S_t + R_t - T_t \sim S_t + R_t$

- Ορίζουμε τα $\bar{D}_t$

$$\bar{D}_t = \frac{1}{n_t} \sum_{j=0}^{n_t - 1} D_{t+jp}, \quad t = s+1, \ldots, p$$

$\bar{D}_t \sim S_t$



$$\bar{D}_t = \frac{1}{n_t} \sum_{j=0}^{n_t - 1} D_{t+(j+1)p}, \quad t = 1, \ldots, s$$

$\bar{D}_1 \sim S_1$, $\bar{D}_2 \sim S_2$, $\bar{D}_3 \sim S_3$

$N = 10$

$\bar{D}_1 = \frac{D_4 + D_7}{2}$, $n_1 = 2$

$\bar{D}_2 = \frac{D_2 + D_5 + D_8}{3}$, $n_2 = 3$

$\bar{D}_3 = \frac{D_3 + D_6 + D_9}{3}$, $n_3 = 3$

$S_t = [S_1, S_2, S_3, S_1, S_2, S_3, S_1, S_2, S_3, S_1]$

- Προσεγγίζουμε τα S_t με τα $\hat{S}_t$

$$\hat{S}_t = \bar{D}_t - \overbrace{\frac{1}{p}\sum_{j=1}^{p}\bar{D}_j}^{\hat{S}^*_t = 0} \sim S_t, \quad t = 1, \dots, p$$

- Επεκτείνουμε σε όλο το μήκος της χρονολογικής σειράς

$$\hat{S}_{t+jp} = \hat{S}_t, \quad j = 1, 2, \dots \quad t = 1, \dots, p$$

Απαλοιφή της εποχικής συνιστώσας

$$Y_t - \hat{S}_t \sim Y_t - S_t = T_t + R_t, \quad t = 1, \ldots, N$$

Παράδειγμα

$$T_t = [10,\ 15,\ 22,\ 24,\ 33,\ 36,\ 40,\ 50,\ 55,\ 55,\ 58,\ 60]^T$$

$$S_t = [10,\ 6,\ 20,\ 10,\ 6,\ 20,\ 10,\ 6,\ 20,\ 10,\ 6,\ 20]^T$$

$$R_t = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^T$$

$$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T$$

Παράδειγμα

$$T_t = [22,\ 27,\ 34,\ 36,\ 45,\ 48,\ 52,\ 62,\ 67,\ 67,\ 70,\ 72]^T$$

$$S_t = [-2,\ -6,\ 8,\ -2,\ -6,\ 8,\ -2,\ -6,\ 8,\ -2,\ -6,\ 8]^T$$

p=3 ←

$$R_t = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^T$$

→ $$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T$$

Παράδειγμα

$$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T, \quad p = 3$$

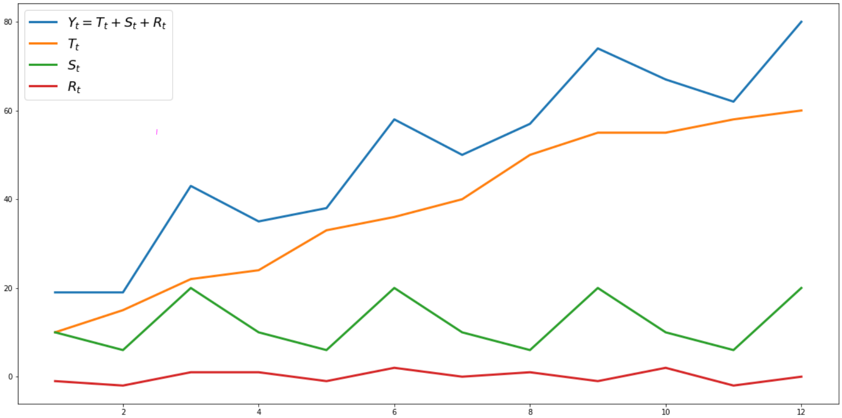
$$Y_t^* = [27,\ 32.3,\ 38.6,\ 43.6,\ 48.6,\ 55,\ 60.3,\ 66,\ 67.6,\ 69.6]^T$$

$$D = [-8,\ 10.6,\ -3.6,\ -5.6,\ 9.3,\ -5,\ -3.3,\ 8,\ -0.6,\ -7.6]^T$$

$$\bar{D}_1 = -3.1, \quad \bar{D}_2 = -6.16, \quad \bar{D}_3 = 9.3$$

$$\hat{S}_1 = -3.13, \quad \hat{S}_2 = -6.18, \quad \hat{S}_3 = 9.31$$

Παράδειγμα



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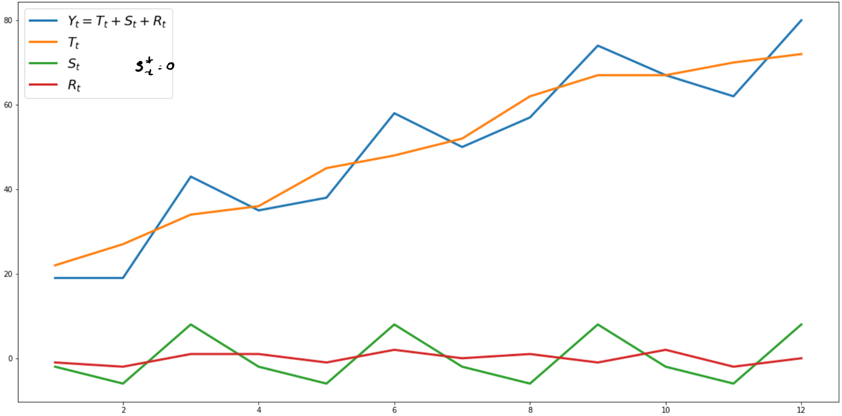

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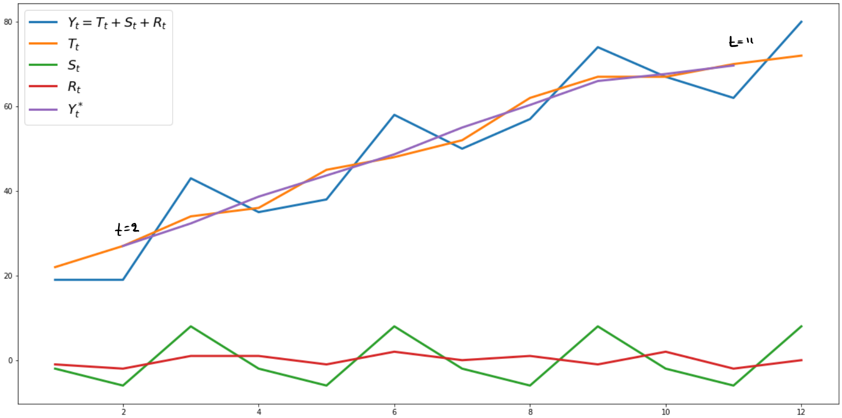

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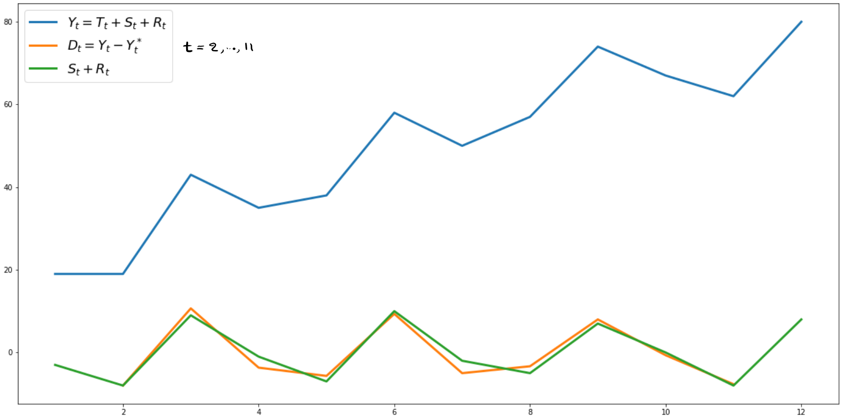

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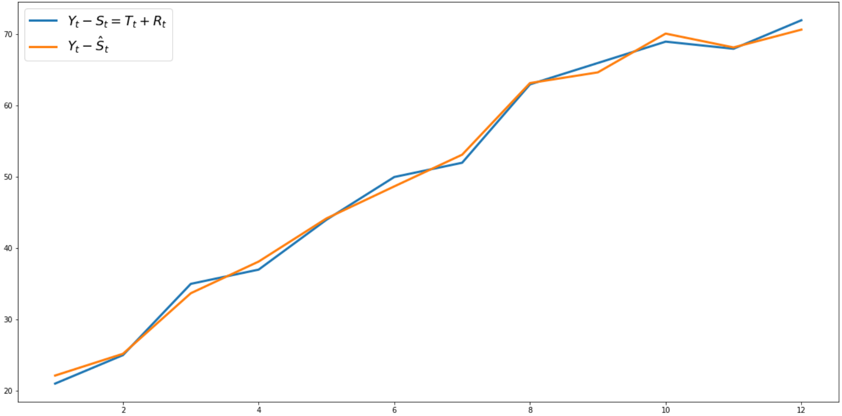


Παράδειγμα

$$T_t = [10,\ 15,\ 22,\ 24,\ 33,\ 36,\ 40,\ 50,\ 55,\ 55,\ 58,\ 60]^T$$

$$S_t = [10,\ 4,\ 5,\ 9,\ 10,\ 4,\ 5,\ 9,\ 10,\ 4,\ 5,\ 9]^T$$

$$R_t = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^T$$

$$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T$$

Δείτε τις σωστές τιμές στην επόμενη διαφάνεια

$s' = 7$

$p = 4$

$\left[\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}\right]^T$

$2s' = 4 \Leftrightarrow s' = 2$

$2s' + 1 = 5$

$\left[\frac{1}{4s}, \frac{1}{2s}, \dots, \frac{1}{2s}, \frac{1}{4s}\right]^T$

Παράδειγμα

$$T_t = [17,\ 22,\ 29,\ 31,\ 40,\ 43,\ 47,\ 57,\ 62,\ 62,\ 65,\ 67]^T$$

$$S_t = [3,\ -3,\ -2,\ 2,\ 3,\ -3,\ -2,\ 2,\ 3,\ -3,\ -2,\ 2]^T$$

$$R_t = [-1,\ -2,\ 1,\ 1,\ -1,\ 2,\ 0,\ 1,\ -1,\ 2,\ -2,\ 0]^T$$

$$Y_t = [19,\ 19,\ 43,\ 35,\ 38,\ 58,\ 50,\ 57,\ 74,\ 67,\ 62,\ 80]^T$$

Δείτε τις σωστές τιμές στη επόμενη διαφάνεια

Παράδειγμα

10 17 28 34 42 42 45 60 64 61 61 69

$$Y_t = [19, 19, 43, 35, 38, 58, 50, 57, 74, 67, 62, 80]^T, \quad p = 4$$

Παράδειγμα

$Y_t =$ Δείτε τις σωστές τιμές στη προηγούμενη διαφάνεια

Θα το ξαναδούμε.

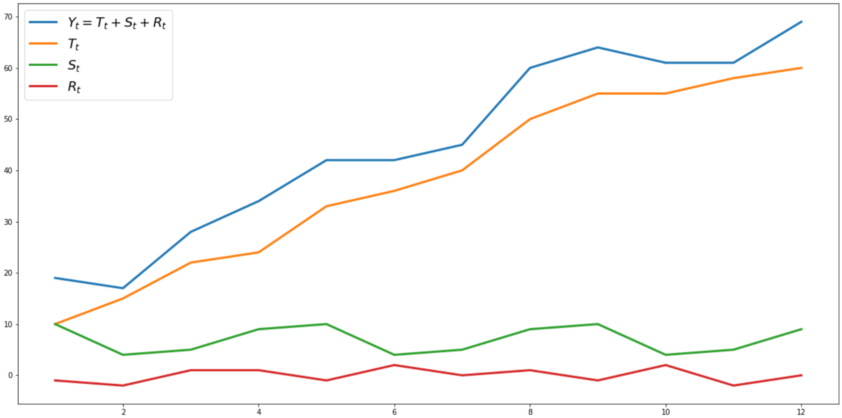
$$Y_t^* = [27.4,\ 33.4,\ 38.6,\ 44,\ 50,\ 55.1,\ 59.5,\ 62.6]^T$$

$$D = [0.6,\ 0.6,\ 0.4,\ -2,\ -5,\ 4.9,\ 4.5,\ -1.6]^T$$

$$\bar{D}_1 = -3.94, \quad \bar{D}_2 = -1.81, \quad \bar{D}_3 = -2.19, \quad \bar{D}_4 = 2.75$$

$$\hat{S}_1 = 3.26, \quad \hat{S}_2 = -2.48, \quad \hat{S}_3 = -2.86, \quad \hat{S}_4 = 2.08$$

Παράδειγμα



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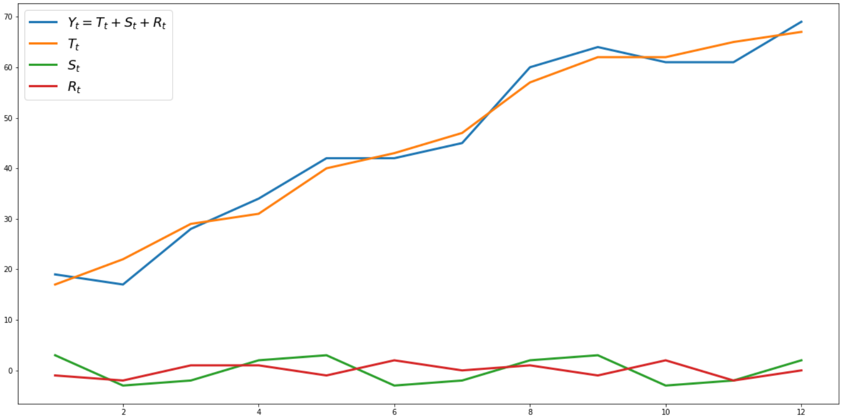

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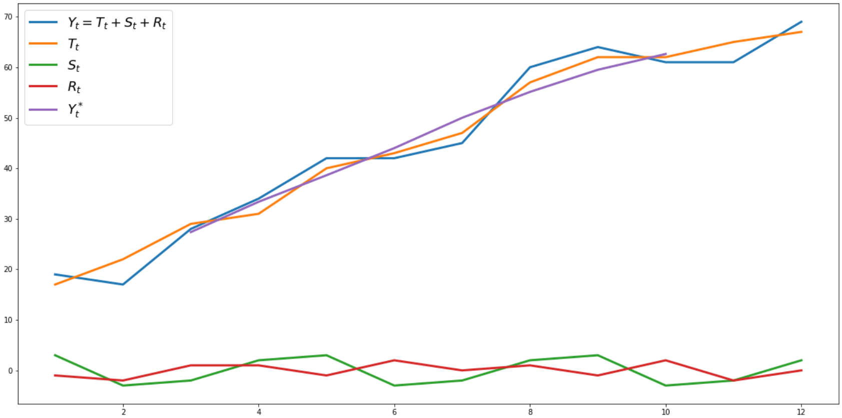

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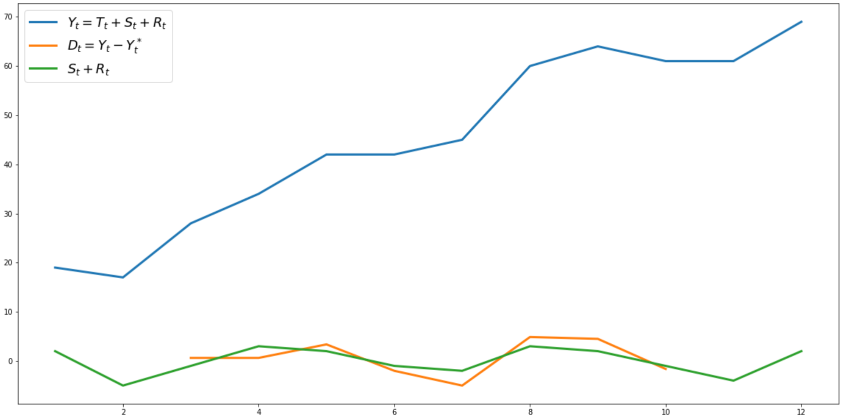

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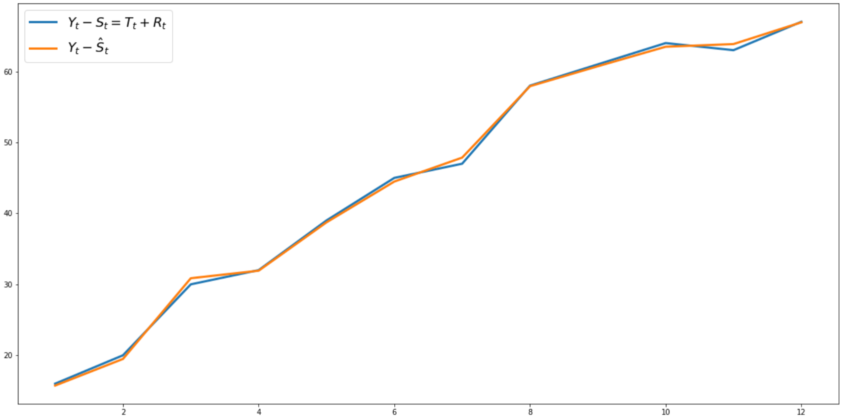

$Y_t \quad t=1,\dots,120 \qquad S_t \qquad p=12 \qquad 2s=12 \Leftrightarrow s=6$

$Y_t^* \quad , \; t=7,\dots,114$

$$Y_t^* = \frac{1}{24} Y_{t-6} + \frac{1}{12}\sum_{u=-5}^{5} Y_{t-u} + \frac{1}{24} Y_{t+6} \quad , \; t=7,\dots,114$$

$$t=7 \qquad Y_7^* = \frac{1}{24} Y_1 + \frac{1}{2}\sum_{u=-5}^{5} Y_{7-u} + \frac{1}{24} Y_{13}$$

$D_t = Y_t - Y_t^* , \; t=7,\dots,114$

$\bar{D}_t \; , \; t=1,\dots,12$

$$\hat{S}_t = \bar{D}_t - E\{\bar{D}_t\}$$

$$E\{\bar{D}_t\} = \frac{1}{12}\sum_{t=1}^{12} \bar{D}_t$$

$Y_t - \hat{S}_t$