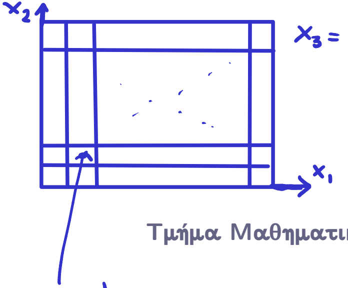
Ασκ 3
Φυλ. 1



$x_3 = 3\,km$

plt.pcolor

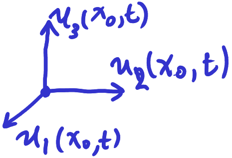
subplot

MEM-284: Κυματική Διάδοση

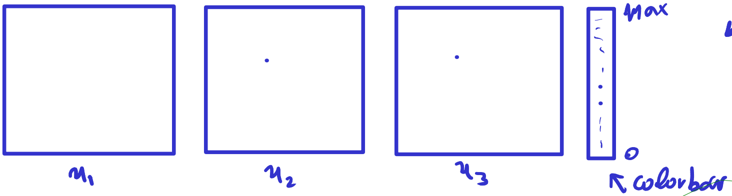
Τμήμα Μαθηματικών και Εφαρμοσμένων Μαθηματικών, Πανεπιστήμιο Κρήτης

u_1 μετατοπισμ στην κατευθυνση του x_1
u_2 » x_2
u_3 x_3

Κώστας Σμαραγδάκης (https://kesmarag.gitlab.io)



7η Διάλεξη - 17.3.2022

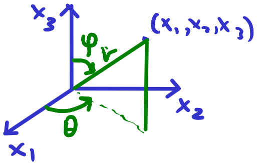


$\delta(\underset{\sim}{x}-\underset{\sim}{x_0})\,\delta(t)$

$i=1,2,3$

$\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases}$

$$\mathcal{G}^S_{i3} = \frac{\delta_{i3} - \gamma_i\gamma_3}{4\pi\rho\beta^2 r}\delta(t - r/\beta)$$



$\begin{bmatrix} g_{13} \\ g_{23} \\ g_{33} \end{bmatrix} =$

- $\dfrac{1}{4\pi\rho\beta^2}[-\gamma_1\gamma_3,\ -\gamma_2\gamma_3,\ 1-\gamma_3^2]^T$: Πρότυπο ακτινοβολίας (radiation pattern).
- $\dfrac{1}{r}$: Εξασθένιση με την απόσταση.
- $\delta(t - r/\beta)$: Οδευών παλμός που απομακρύνεται με ταχύτητα β.

$(x_1, x_2, x_3) \to (r, \vartheta, \varphi)$

$x_1 = r\sin\varphi\cos\vartheta$

$x_2 = r\sin\varphi\sin\vartheta$

$x_3 = r\cos\phi$

$$\begin{aligned} \gamma_1 &= \sin\phi\cos\theta, \quad \theta\in[0,2\pi),\ \phi\in[0,\pi] \\ \gamma_2 &= \sin\phi\sin\theta \\ \gamma_3 &= \cos\phi \end{aligned}$$

$\gamma_i = \dfrac{x_i}{r}, \ r>0$

$$g^S_{i3} = \frac{\delta_{i3}-\gamma_i\gamma_3}{4\pi\rho\beta^2}\cdot\frac{1}{r}\cdot\delta(t-r/\beta)$$

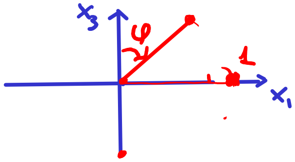
$$\left[-\gamma_1\gamma_3,\ -\gamma_2\gamma_3,\ 1-\gamma_3^2\right] = \underset{\sim}{c}$$

$$\|\underset{\sim}{c}\|_2^2 = \sin^2\phi\cos^2\theta\cos^2\phi + \sin^2\phi\sin^2\theta\cos^2\phi + \left(1-\cos^2\phi\right)^2$$

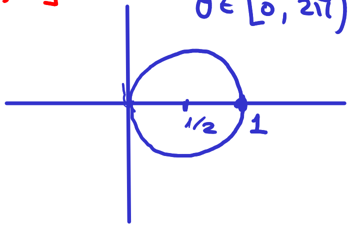
$$= \sin^2\phi\cos^2\phi\left(\cos^2\theta+\sin^2\theta\right) + \sin^4\phi =$$

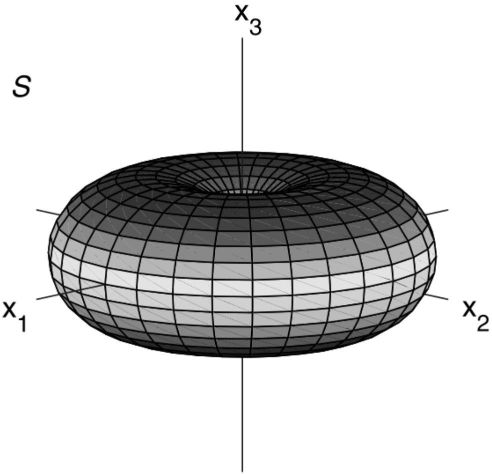
$$= \sin^2\phi\cos^2\phi + \sin^4\phi = \sin^2\phi\left(\cos^2\phi+\sin^2\phi\right) = \sin^2\phi$$

$$\|\underset{\sim}{c}\|_2 = |\sin\phi| \qquad \theta = 0 \qquad \phi\in[0,\pi]$$

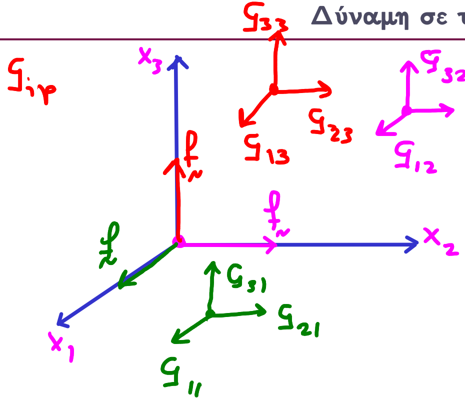


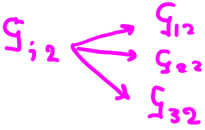
$\theta\in[0,2\pi)$



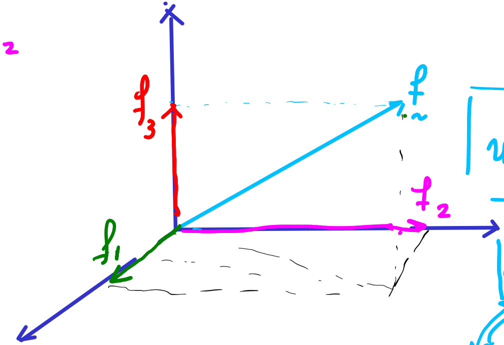

x3
S
x1
x2

Δύναμη σε τυχαίο προσανατολισμό





$$\underset{\sim}{f} = (f_1, f_2, f_3)$$



$$\boxed{u_i = \sum_{q=1}^{3} f_q * G_{iq}}$$

$$\Updownarrow$$

$$\boxed{u_i = f_q * G_{iq}}$$

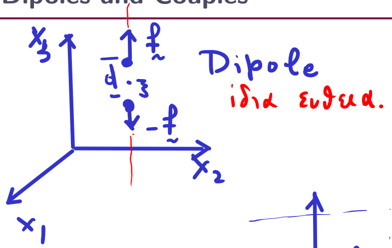
$$u_i = f_1 * G_{i1} \quad , \; i = 1,2,3$$

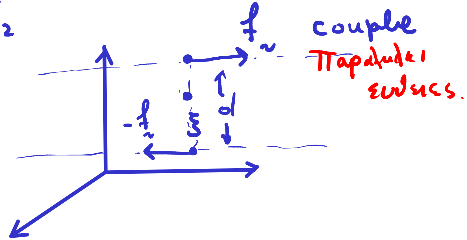
$$u_i = f_2 * G_{i2}$$

$$u_i = f_3 * G_{i3}$$

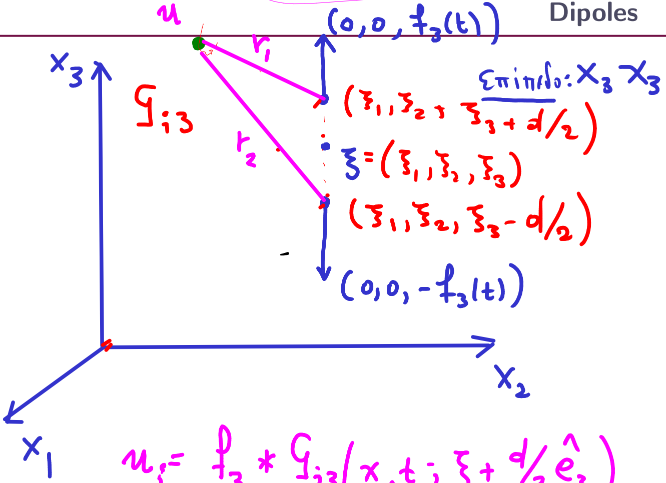
$$u_i = f_1 * G_{i1} + f_2 * G_{i2} + f_3 * G_{i3}$$

$i = 1,2,3$





- Ζεύγος παράλληλων Δυνάμεων.
- Σημεία εφαρμογής σε μικρή απόσταση.
- Αντίθετη φορά



$(x_1=0)$

$(0,0,f_3)$: $G_{i3} * f_3$ — $G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi}+d/2\,\hat{e}_3)$

$(0,0,-f_3)$: $-G_{i3} * f_3$ — $G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi}-d/2\,\hat{e}_3)$

$$u_i = f_3 * G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi}+d/2\,\hat{e}_3) - f_3 * G(\underset{\sim}{x},t;\underset{\sim}{\xi}-d/2\,\hat{e}_3) = f_3\, d\, \frac{G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi}+d/2\,\hat{e}_3)-G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi}-d/2\,\hat{e}_3)}{d}$$

$d\to 0$, $f_3\to\infty$ s.t. $d\cdot f_3 = M_{33} < \infty$

$$= M_{33}\frac{\partial}{\partial \xi_3} G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi})$$

Άσκηση 2 φυλ.

$$u_{xx} - c^{-2} u_{tt} + f(x) = 0 \qquad x > 0$$

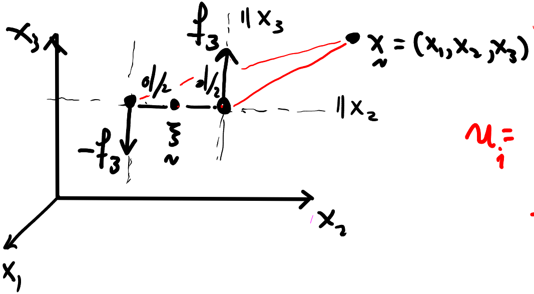
$$u(x,0) = g(x), \quad u_t(x,0) = h(x)$$

$$+ \quad u(0,t) = 0$$

$$\rightarrow \quad x \in \mathbb{R}. \qquad \tilde{f} = \begin{cases} f, & x > 0 \\ 0, & x = 0 \\ -f, & x < 0 \end{cases}$$

$$\tilde{g} = \begin{cases} g, & x > 0 \\ 0, & x = 0 \\ -g, & x < 0 \end{cases} \qquad h = \begin{cases} h \\ 0 \\ -h \end{cases}$$

$$u = \tilde{u}\Big|_{x>0} \qquad \tilde{u} \quad x \in \mathbb{R}$$



$$u_i = f_3 * G_{i3}(\underset{\sim}{x}, t; \underset{\sim}{\xi} + d/2\, \hat{e}_2) - f_3 * G_{i3}(\underset{\sim}{x}, t; \underset{\sim}{\xi} - d/2\, \hat{e}_2)$$

$$u_i(\underset{\sim}{x},t) = d f_3 * \frac{G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi} + d/2\,\hat{e}_2) - G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi} - d/2\,\hat{e}_2)}{d}$$

$$\left.\begin{array}{l} f_3 \to \infty \\ d \to 0 \end{array}\right\} \quad d f_3 = M_{32} < \infty$$

$$u_i(\underset{\sim}{x},t) = M_{32} * \frac{\partial G_{i3}(\underset{\sim}{x},t;\underset{\sim}{\xi})}{\partial \xi_2}$$

<u>Γενικά</u>: $\underset{\approx}{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$

$$u_i = M_{pq} * \frac{\partial G_{ip}}{\partial \xi_q} = \sum_{p=1}^{3}\sum_{q=1}^{3} M_{pq} * \frac{\partial G_{ip}}{\partial \xi_q}$$