MEM-284: Κυματική Διάδοση

Τμήμα Μαθηματικών και Εφαρμοσμένων Μαθηματικών, Πανεπιστήμιο Κρήτης

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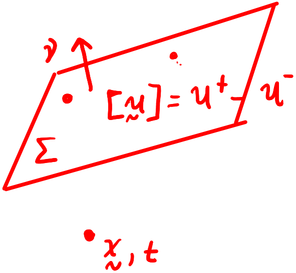


10η Διάλεξη - 7.4.2022

Θεώρημα αναπαράστασης

$$u_n(\mathbf{x}, t) = \int_{\Sigma_\xi} \nu_j c_{ijpq}[u_i](\boldsymbol{\xi}, t) * \frac{\partial}{\partial \xi_q} \mathcal{G}_{np}(\mathbf{x}, t; \boldsymbol{\xi}) d\Sigma_\xi$$

Ισοδύναμη δύναμη

$$e_p(\boldsymbol{\eta}, t) = -\int_{\Sigma_\xi} \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \boldsymbol{\xi}) \nu_j c_{ijpq}[u_i](\boldsymbol{\xi}, t) d\Sigma_\xi$$

Ομοιόμορφα και ισοτροπικά ελαστικά μέσα

$$c_{ijpq} = \lambda \delta_{ij} \delta_{pq} + \mu(\delta_{ip}\delta_{jq} + \delta_{iq}\delta_{jp})$$

Άσκηση 1

Για ομοιόμορφο και ισοτροπικό μέσο υπολογίστε τις σταθερές c_{13pq}

$$c_{13pq} = \lambda \cancel{\delta_{13}}^{\,0} \delta_{pq} + \mu \left(\delta_{1p}\delta_{3q} + \delta_{1q}\delta_{3p} \right)$$

$$= \begin{cases} \mu, & p=1, q=3 \text{ ή } p=3, q=1 \\ 0, & \text{διαφορετικά.} \end{cases}$$

Για ομοιόμορφο και ισοτροπικό μέσο υπολογίστε τις σταθερές c_{33pq}

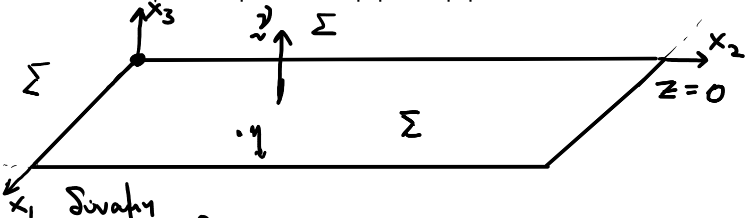
$$c_{33pq} = \lambda \overset{1}{\cancel{\delta_{33}}} \delta_{pq} + \mu \left(\delta_{3p}\delta_{3q} + \delta_{3q}\delta_{3p} \right) =$$

$$= \lambda \delta_{pq} + 2\mu \delta_{3p}\delta_{3q} = \begin{cases} \lambda, & p = q \neq 3 \\ \lambda + 2\mu, & p = q = 3 \\ 0, & \text{διαφορετικά.} \end{cases}$$

Θεωρήστε άπειρο ομοιόμορφο και ισοτροπικό ελαστικό μέσο. Για μια εξάρθρωση (dislocation) που συμβαίνει στο επίπεδο $z = 0$ που περιγράφεται ως

$$[u_1] = \delta(x_1)\delta(x_2)H(t), \ [u_2] = [u_3] = 0 \qquad z=0$$

Υπολογίστε την ισοδύναμη δύναμη.



Δύναμη

$$e_p(\eta,t) = -\int_{\Sigma_\xi} \frac{\partial}{\partial \eta_q}\delta(\eta - \xi)\,\nu_j\, c_{ijpq}\,[u_i](\xi,t)\,d\Sigma_\xi \,, \quad p \in \{1,2,3\}$$

$\nu = (0,0,1) \rightarrow \nu_j = \delta_{j3} \qquad [u_1] \neq 0 \qquad [u_2] = [u_3] = 0$

$$e_p(\tilde{\eta},t) = -\int_{\Sigma_{\tilde{\xi}}} \frac{\partial}{\partial \eta_q}\delta(\tilde{\eta}-\tilde{\xi})\,\nu_3\, \underline{C_{13pq}}[u_1](\tilde{\xi},t)\, d\Sigma_{\tilde{\xi}} =$$

$$C_{13pq} = \begin{cases} \mu, & p=1,\ q=3 \ \text{ή}\ p=3,\ q=1 \\ 0, & \text{διαφορετικά} \end{cases}$$

$$= -\int_{\mathbb{R}}\int_{\mathbb{R}} \frac{\partial}{\partial \eta_q}\delta(\tilde{\eta}-\tilde{\xi})\, C_{13pq}\boxed{\delta(\xi_1)\delta(\xi_2)}H(t)\, d\Sigma_{\tilde{\xi}} =$$

$$= -\frac{\partial}{\partial \eta_q}\left\{\delta(\eta_1)\delta(\eta_2)\delta(\eta_3)\right\} C_{13pq}\, H(t) \Rightarrow$$

$$\tilde{e} = \left(-\frac{\partial}{\partial \eta_3}\left\{\delta(\eta_1)\delta(\eta_2)\delta(\eta_3)\right\} C_{1313}\, H(t),\ 0,\ -\frac{\partial}{\partial \eta_1}\left\{\delta(\eta_1)\delta(\eta_2)\delta(\eta_3)\right\} C_{1331}\, H(t)\right)$$

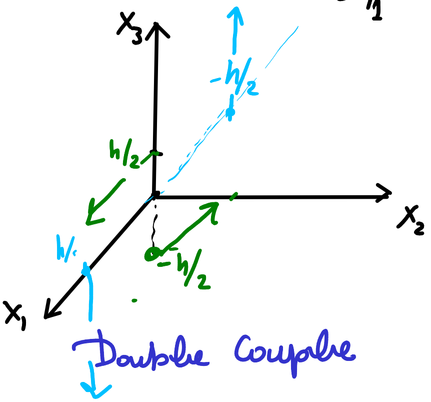
$$\underset{\sim}{e} = \left(-\mu H(t)\,\delta(\eta_1)\delta(\eta_2)\frac{\partial}{\partial \eta_3}\delta(\eta_3),\, 0,\, -\mu H(t)\,\delta(\eta_2)\delta(\eta_3)\frac{\partial}{\partial \eta_1}\delta(\eta_1) \right)$$

$\underline{p=1}$ Couple.

$$\frac{\partial}{\partial \eta_3}\delta(\eta_3) = \lim_{h\to 0} \frac{\delta(\eta_3 + h/2) - \delta(\eta_3 - h/2)}{h}$$

$\underline{p=3}$ Couple

$$\frac{\partial}{\partial \eta_1}\delta(\eta_1) = \lim_{h\to 0} \frac{\delta(\eta_1 + h/2) - \delta(\eta_1 - h/2)}{h}$$



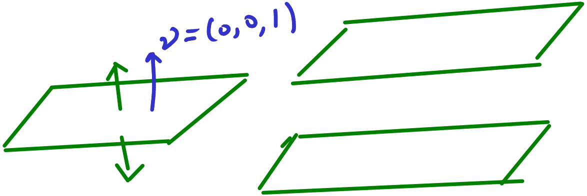
Double Couple

Άσκηση 4

Θεωρήστε άπειρο ομοιόμορφο και ισοτροπικό ελαστικό μέσο. Για μια εξάρθρωση (dislocation) που συμβαίνει στο επίπεδο $z = 0$ που περιγράφεται ως

$$[u_1] = [u_2] = 0, \ [u_3] = \delta(x_1)\delta(x_2)H(t)$$

Υπολογίστε την ισοδύναμη δύναμη.



$i = 3 = j$

$$e_p(\underset{\sim}{\eta},t) = -\int_{\Sigma_\xi} \frac{\partial}{\partial \eta_q}\delta(\underset{\sim}{\eta}-\underset{\sim}{\xi})\,\nu_j\, c_{ijpq}\,[u_i](\underset{\sim}{\xi},t)\,d\Sigma_{\underset{\sim}{\xi}}$$

c_{33pq}

$$e_p(\underset{\sim}{\eta}, t) = -\frac{\partial}{\partial \eta_q}\{\delta(\eta_1)\delta(\eta_2)\delta(\eta_3)\} C_{33pq}[u_3].$$

$$p = q \begin{cases} = 3 \\ \neq 3 \end{cases}$$

$p=1$ $\quad \frac{\partial}{\partial \eta_1}$ $\qquad\qquad$ $p=2$ $\quad \frac{\partial}{\partial \eta_2}$

$$\underset{\sim}{e} = \Big(-\lambda \frac{\partial}{\partial \eta_1}\{\delta(\eta_1)\}\delta(\eta_2)\delta(\eta_3)H(t), -\lambda\frac{\partial}{\partial \eta_2}\{\delta(\eta_2)\}\delta(\eta_1)\delta(\eta_3)H(t),$$

$$-(\lambda+2\mu)\frac{\partial}{\partial \eta_3}\{\delta(\eta_3)\}\delta(\eta_1)\delta(\eta_2)H(t)\Big)$$

$p=3$ $\quad \frac{\partial}{\partial \eta_3}$