Για απορίες 04-06 19:00.

Linear Regression

$$\{(0,0,0),\ (1,1,5),\ (2,0,5),\ (1,2,8)\}$$

length 4

$$Y = A + B^{(1)} X^{(1)} + B^{(2)} X^{(2)} + \varepsilon$$

$$\varepsilon \sim N(0, \sigma_\varepsilon^2)$$

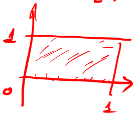
$$\underset{\approx}{X} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \qquad \underset{\sim}{y} = \begin{bmatrix} 0 \\ 5 \\ 5 \\ 8 \end{bmatrix} \qquad \underset{\approx}{X}^T \underset{\approx}{X} = \begin{bmatrix} 4 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 5 \end{bmatrix}$$

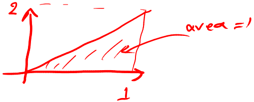
$$(\underset{\approx}{X}^T \underset{\approx}{X}) \underset{\sim}{\beta} = \underset{\approx}{X}^T \underset{\sim}{y} \qquad \underset{\approx}{X}^T \underset{\sim}{y} = \begin{bmatrix} 18 \\ 23 \\ 21 \end{bmatrix}$$

$$\hat{y} = \underbrace{-0.045}_{a} + \underbrace{2.5}_{b^{(1)}} x^{(1)} + \underbrace{2.73}_{b^{(2)}} x^{(2)}$$

$H_0: f_0(x) = 1$, $x \in (0,1)$ vs $H_1: f_1(x) = 2x$, $x \in (0,1)$

$U[0,1]$





Size — probability of Rejecting the null hypothesis.

$$\phi(x) = \begin{cases} 1, & \Lambda(x) > k \\ 0, & \Lambda(x) \le k \end{cases}, \quad k \in (0,2)$$

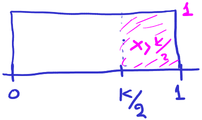
test function

$k/2 \in (0,1)$

Neyman-Pearson

Size:

$$\Lambda(x) = \frac{f_1(x)}{f_0(x)} = \frac{2x}{1} = 2x, \quad x \in (0,1)$$



under H_0

$$\mathbb{E}_{H_0}\{\phi(X)\} = 1 \cdot P_{H_0}\{\Lambda(X) > k\} = P_{H_0}\{2X > k\}$$
$$= P_{H_0}(X > k/2) = 1 - k/2$$

Power: $\mathbb{E}\{\phi(X)\}$ under the alternative Hypothesis.

$$\mathbb{E}_{H_1}\{\phi(X)\} = 1 \cdot P_{H_1}(\Lambda(X) > k) = P_{H_1}\left(X > \frac{k}{2}\right) = 1 - P_{H_1}\left(X \leq \frac{k}{2}\right)$$

$$= 1 - \int_0^{k/2} 2x \, dx =$$

$$= 1 - x^2 \Big|_0^{k/2} =$$

$$= 1 - \frac{k^2}{4}$$

$$\underset{k}{\operatorname{argmax}} \left\{ \mathbb{E}_{H_1}\{\phi(X)\} - \mathbb{E}_{H_0}\{\phi(X)\} \right\}$$

t-test

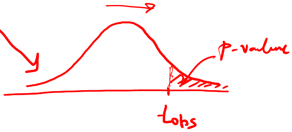
X_1, X_2 i.i.d $\quad X_1 \sim N(\mu, 2)$

$$T = \frac{\bar{X}_2 - \mu_0}{S_2/\sqrt{n}}$$

$H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$

$H_1: \mu \neq \mu_0$

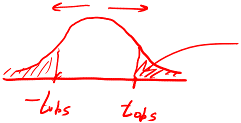
$$T \sim T(df = 2-1 = 1)$$



$df >> 1 \longrightarrow N(0,1)$

example

$$\boxed{\begin{array}{l} x = (0, 1) \\ \mu_0 = 0 \end{array}}$$



$2P(T > t_{obs})$

$$\bar{x}_2 = \frac{1}{2}$$

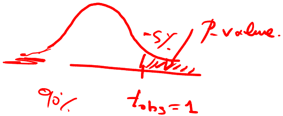
$$S_2^2 = \frac{(0-\frac{1}{2})^2 + (1-\frac{1}{2})^2}{2-1} = 2 \cdot \frac{1}{4} = \frac{1}{2}$$

✗ 2way. ANOVA

✗ logistic Regression

$$t_{obs} = \frac{\frac{1}{2} - 0}{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$H_1 : \mu > 0$



$$\text{P-value} = 1 - P(T \leq t_{obs})$$

$$T \sim \mathcal{T}(df = 1)$$