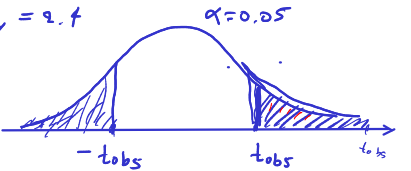
Agenom 1 dw. 2

120 $\begin{cases} 60 & n_x = 60 \quad \bar{x}_{m_x} = 26.4 \\ 60 & n_y = 60 \quad \bar{y}_{m_y} = 18.6 \end{cases}$

$S_{\bar{x}_{m_x} - \bar{y}_{m_y}} = 2.4$



$$t_{obs} = \frac{\bar{x}_{m_x} - \bar{y}_{m_y}}{S_{\bar{x}-\bar{y}}} = \frac{26.4 - 18.6}{2.4} \dots$$

$n_x + n_y - 2 = 118$

$H_0: \mu_x = \mu_y$ vs $H_1: \mu_x \neq \mu_y$

$\mu_x > \mu_y$ ή $\mu_x < \mu_y$

$T \sim T(df = 118)$

$H_0: \mu_x = \mu_y$ vs $H_1: \mu_x > \mu_y$

Answer: X_1, X_2 iid $\quad f(x_j;\theta) = \left(\frac{1}{\theta}\right)^{x_j}\left(\frac{\theta-1}{\theta}\right)^{1-x_j}$ $\quad x_j \in \{0,1\}$, $\theta \in [2,\infty)$

$$f(x_j;\theta) = \begin{cases} 1/\theta & x_j = 1 \\ \frac{\theta-1}{\theta}, & x_j = 0 \end{cases}$$

$H_0: \theta = 2$, $H_1: \theta > 2$

θ_0 $\qquad$ θ_1

ϕ_0 test function UMP of size 0.05

$\underset{\sim}{x} = (x_1, x_2)^T$

$$f(\underset{\sim}{x};\theta) = \left(\frac{1}{\theta}\right)^{x_1+x_2}\left(\frac{\theta-1}{\theta}\right)^{2-(x_1+x_2)}$$

$\theta_1 \in \Theta_1$ $\qquad$ $\theta_0 \in \Theta_0$

$$\Lambda(\underset{\sim}{x}) = \frac{f(\underset{\sim}{x};\theta_1)}{f(\underset{\sim}{x};\theta_0)} = \frac{\left(\frac{1}{\theta_1}\right)^{x_1+x_2}\left(\frac{\theta_1-1}{\theta_1}\right)^{2-(x_1+x_2)}}{\left(\frac{1}{\theta_0}\right)^{x_1+x_2}\left(\frac{\theta_0-1}{\theta_0}\right)^{2-(x_1+x_2)}} = \left(\frac{\theta_0}{\theta_1}\right)^{x_1+x_2+2-(x_1+x_2)}\left(\frac{\theta_1-1}{\theta_0-1}\right)^{2-(x_1+x_2)}$$

$$= \left(\frac{\theta_0}{\theta_1}\right)^2\left(\frac{\theta_1-1}{\theta_0-1}\right)^{2-(x_1+x_2)} \overset{\theta_0=2}{=\!=} \left(\frac{2}{\theta_1}\right)^2 (\theta_1-1)^{2-(x_1+x_2)}$$

$t(\underset{\sim}{x}) = -(x_1+x_2)$ $\qquad$ $\tilde{\Lambda}(t(\underset{\sim}{x})) = \left(\frac{2}{\theta_1}\right)^2(\theta_1-1)^2 + t(\underset{\sim}{x})$

ϕ_0 UMP test function.

$$\phi_0(\underset{\sim}{x}) = \begin{cases} 1, & t(\underset{\sim}{x}) > t_0 \\ \gamma, & t(\underset{\sim}{x}) = t_0 \\ 0, & t(\underset{\sim}{x}) < t_0 \end{cases}$$

$x_1, x_2 \in \{0,1\}$

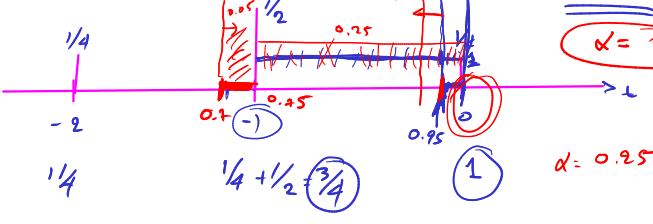
$t(\underset{\sim}{x}) \in \{-2, -1, 0\}$

$x_1 = x_2 = 1$; $x_1 = 1, x_2 = 0$ or $x_1 = 0, x_2 = 1$; $x_1 = x_2 = 0$

$f(t=0;\theta) = f(\underset{\sim}{x}=(0,0);\theta) = \left(\frac{\theta-1}{\theta}\right)^2$ under H_0 $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$f(t=-1;\theta) = f(\underset{\sim}{x}=(1,0);\theta) + f(\underset{\sim}{x}=(0,1);\theta) = 2 f(\underset{\sim}{x}=(1,0);\theta) = 2\,\frac{1}{\theta}\,\frac{\theta-1}{\theta} \overset{H_0}{=} \frac{1}{2}$

$f(t=-2;\theta) = f(\underset{\sim}{x}=(1,1);\theta) = \left(\frac{1}{\theta}\right)^2 \overset{H_0}{=} \frac{1}{4}$ Reject



$\alpha = 0.05$ $\alpha = 0.1$ $\alpha = 0.3$

$\gamma = \frac{1-0.9}{1-0.25} = 0.4$

$\alpha = 0.25$

F 1/4 1/4 + 1/2 = (3/4) (1)

$$\phi_0 = \begin{cases} \gamma, & t = t_0 = 0 \\ 0, & t < 0 \end{cases} \qquad \gamma = \frac{1 - 0.95}{1 - 0.75} = 0.2$$

UMPT $\qquad \phi_0(\underset{\sim}{x}) = \begin{cases} 0.2, & x_1 = x_2 = 0 \\ 0, & \text{allrwise} \end{cases}$

$$E_{\theta_0}[\phi_0(X)] = 0.2 \cdot f(x_1 = x_2 = 0) = \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20} = 0.05$$

Si $\alpha = 0.3$

$$\gamma = \frac{0.75 - 0.7}{0.75 - 0.25} = \frac{0.05}{0.5} = 0.1$$

$$\phi_0(\underset{\sim}{x}) = \begin{cases} 1, & \underset{\sim}{x} = (0,0) \\ 0.1, & \underset{\sim}{x} = (0,1) \text{ ou } \underset{\sim}{x} = (1,0) \\ 0, & \text{autrwise} \end{cases}$$

H_0 true τότε η πιθανότητα να κάνω λάθος θα ήταν 0.3

Άσκηση 2 φυλ. 9

	Y_{11}	Y_{12}		
C_1:	47.62	49.79	→	$\bar{Y}_{1,2} = 48.705$
C_2:	40.45	43.46	→	$\bar{Y}_{2,2} = 41.955$
C_3:	21.25	22.34	→	$\bar{Y}_{3,2} = 21.795$
C_4:	13.18	11.65	→	$\bar{Y}_{4,2} = 12.415$
C_5:	8.51	8.13	→	$\bar{Y}_{5,2} = 8.32$

Overall Sample mean

$$\bar{\bar{Y}} = 26.638$$

S_p^2

S_b^2

$$S_{i,2}^2 = \frac{1}{2-1}\sum_{j=1}^{2} e_{ij}^2 \qquad e_{ij} = Y_{ij} - \bar{Y}_{i,2}$$

$S_{1,2}^2 = 2.35445$

$S_{2,2}^2 = 4.53005$

$S_{3,2}^2 = 0.59405$

$S_{4,2}^2 = 1.17045$

$S_{5,2}^2 = 0.0722$

Pooled Sample variance

$$S_p^2 = \frac{\sum (n_i - 1) S_{i,n_i}^2}{\sum (n_i - 1)} = \frac{\sum S_{i,2}^2}{\sum 1} = \frac{\sum S_{i,2}^2}{5} = 2.907$$

$df_p = 5$

$$S_b^2 = \frac{\sum n_i (\bar{X}_{i,n_i} - \bar{X})^2}{I - 1} = \frac{\frac{1}{2}\sum 2 (\bar{X}_{i,2} - \bar{X})^2}{4}$$

$$= \frac{1}{2}\{ (48.705 - 26.638)^2 + (41.955 - 26.638)^2 + (21.705 - 26.638)^2 + (12.415 - 26.638)^2 + (8.32 - 26.638)^2 \}$$

$df_b = 4$

$$F_{obs} = \frac{S_b^2}{S_p^2} \sim F(4, 5)$$