$X \sim Bin(100, \theta)$, $\theta \in [0.5, 1)$

$H_0 : \theta = 0.5$ vs $H_1 : \theta \in (0.5, 1)$

$\theta = 0.5$ $\quad$ $\theta \in (0, 0.5) \cup (0.5, 1)$

x a realization of X , $x \in \{0, 1, \dots, 100\} = \mathcal{X}$ ← Sample space

$x = 60$ ← observation

Probability mass function (pmf) - συνάρτηση μάζας πιθανότητας

$$f(x;\theta) = \binom{100}{x} \theta^x (1-\theta)^{100-x} = P\{X = x\}$$

if H_0 is true

$E\{X\} = 50$

$Var\{X\} = 25$

$E\{X\} = N\theta$

$Var\{X\} = N\theta(1-\theta)$

if $\max\{N\theta, N(1-\theta)\} > 5$ then

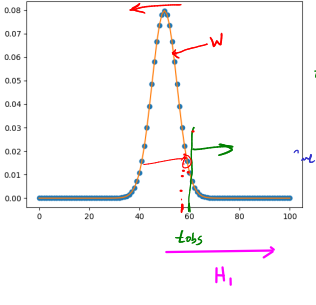
$X \to W \sim N\left(N\theta, N\theta(1-\theta)\right)$

$T(X) = X$

$t_{obs} = 60$

Under H_0 $\quad$ $f(x; 0.5) = \binom{100}{x}\left(\frac{1}{2}\right)^{100}$

$$P\text{-value} = \sum_{x=60}^{100} \binom{100}{x}\left(\frac{1}{2}\right)^{100}$$



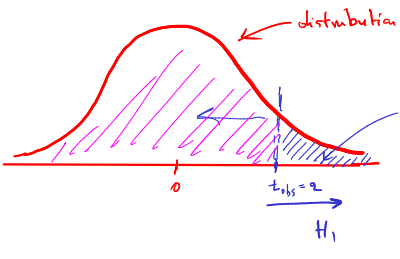
An alternative approach

Under H_0

$$T(X) = \frac{X - \mathbb{E}(X)}{\sqrt{Var(X)}} = \frac{X - 50}{5} \rightarrow \frac{W - 50}{5} \qquad W \sim N(50, 25)$$

$$T \sim N(0, 1)$$

observation: $X = 60$ $\qquad t_{obs} = \frac{60 - 50}{5} = \frac{10}{5} = 2$ ← z-score



Search on Google z-score calculator !!!

$$p\text{-value} = 1 - \mathbb{P}(T \leq t_{obs}) = 1 - 0.97725 =$$

$$= 0.02275$$