MEM-264 Applied Statistics

Department of Mathematics and Applied Mathematics, University of Crete

Costas Smaragdakis (kesmarag@uoc.gr)

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* We need the squared sum of the grades * $\sum_{i,j,k} Y_{ijk}^2 = 82000$

$Y_{ijk} = \mu + \alpha_i + b_j + \gamma_{ij} + \varepsilon_{ijk}$, $\varepsilon_{ijk} \sim N(0, \sigma^2)$

Example : Student's Grades

I

Department	Male (j=1)	Female (j=2)	
Mathematics (i=1)	310 (mean: 6.38)	400 (mean: 6.52)	710 (mean: 6.46)
Physics (i=2)	510 (mean: 6.41)	330 (mean: 6.56)	840 (mean: 6.47)
Computer Science (i=3)	190 (mean: 6.44)	210 (mean: 6.41)	400 (mean: 6.42)
	1010 (mean: 6.41)	940 (mean: 6.51)	**1950 (mean: 6.46)**

J

3 ANOVA TESTS

Pooled Sample variance $S_p^2 = \frac{\sum_{i,j} (n_{ij} - 1) S_{ij,n_{ij}}^2}{N - IJ} = \frac{\sum_{ij,k} (Y_{ijk} - \overline{Y}_{ij,n_{ij}})^2}{N - IJ}$

$\bar{Y}_{ij} = \bar{Y}_{ij,n_{ij}}$

$$\sum_k \left(Y_{ijk} - \bar{Y}_{ij} \right)^2 = \sum_k Y_{ijk}^2 - 2\bar{Y}_{ij} \sum_k Y_{ijk} + n_{ij}\bar{Y}_{ij}^2 =$$

$$= \sum_k Y_{ijk}^2 - 2n_{ij}\bar{Y}_{ij}^2 + n_{ij}\bar{Y}_{ij}^2$$

$$= \sum_k Y_{ijk}^2 - (n_{ij}-1)\bar{Y}_{ij}^2 \qquad \sum_{ijk} (Y_{ijk} - \bar{Y}_{ij})^2 = \sum_{i,j,k} Y_{ijk}^2 - \sum_{i,j} (n_{ij}-1)\bar{Y}_{ij}^2$$

$$S_p^2 = \frac{89000 - 309 \cdot 6.38^2 - 399 \cdot 6.58^2 - \dots - 209 \cdot 6.41^2}{1944}$$

$$S_b^2 = \frac{SSM}{DF} = \frac{\sum_{i,j} (\bar{Y}_{ij,n_{ij}} - \bar{Y})^2}{IJ-1}$$

$$SSM = SSI + SSJ + SSIJ$$

$$DF = DFI + DFJ + DFIJ$$

$$IJ-1 = I-1 + J-1 + (I-1)(J-1)$$

$$F_I = \frac{SSI/DFI}{S_p^2}$$

$$F_J = \frac{SSJ/DFJ}{S_p^2}$$

$$F_{IJ} = \frac{SSIJ/DFIJ}{S_p^2} = \frac{(SSM - SSI - SSJ)/DFIJ}{S_p^2}$$

H_{I0}

$F_I \sim F(2, 1944)$

$F_J \sim F(1, 1944)$

$F_{IJ} \sim F(2, 1944)$

$(I-1)(J-1)$

$$SSI = \sum_i (\bar{Y}_{i,n_i} - \bar{Y})^2 = (6.46-6.46)^2 + (6.47-6.46)^2 + (6.42-6.46)^2 \qquad DFI = I-1 = 2$$

$$SSJ = \sum_j (\bar{Y}_{j,n_j} - \bar{Y})^2 = (6.41-6.46)^2 + (6.51-6.46)^2 \qquad DFJ = J-1 = 1$$

$$SSIJ = SSM - SSI - SSJ$$

I

$H_{I0}: \alpha_1 = \alpha_2 = \cdots \alpha_I = 0$

J

$H_{J0}: b_1 = b_2 = \cdots b_J = 0$

IJ

$H_{IJ0}: \gamma_{11} = \gamma_{12} \cdots = 0$

$SSM = (6.38 - 6.96)^2 + (6.52 - 6.96)^2 + (6.41 - 6.96)^2 + \cdots + (6.41 - 6.96)^2$