

MEM-264 Applied Statistics

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The F-Test

I - Groups

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_I \quad \text{vs} \quad H_1 : \mu_\ell \neq \mu_m \quad \text{for at least one pair of indices } (\ell, m).$$

$n_1 \quad n_2 \quad n_I \quad N = \sum_{i=1}^I n_i$

F statistics

$$F = \frac{S_b^2}{S_p^2}$$

$$S_b^2 = \frac{1}{I-1} \sum_{i=1}^I (\bar{Y}_{i,n_i} - \bar{Y})^2$$

Sample mean of the i -th group

Sample mean (overall)

$$S_p^2 = \frac{\sum_{i=1}^I (n_i - 1) S_{i,n_i}^2}{\sum_{i=1}^I (n_i - 1)} \leftarrow N - I$$

Sample variance of the i -th group

- ▶ If H_0 is true then $F \approx 1$
- ▶ If H_0 is false then $F > 1$

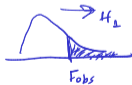
F-distribution

$$\underbrace{y_{11}, y_{12}, \dots, y_{1n_1}}_{\text{GROUP 1}}, \underbrace{y_{21}, \dots, y_{2n_2}}_{\text{GROUP 2}}, \dots, \underbrace{y_{I1}, \dots, y_{In_I}}_{\text{GROUP I}} \rightarrow N$$

$H_0 : \mu_1 = \mu_2 = \dots = \mu_I$ vs $H_1 : \mu_\ell \neq \mu_m$ for at least one pair of indices (ℓ, m) .

$$F = \frac{S_b^2}{S_p^2} \quad F \sim \tilde{F}(I-1, N-I)$$

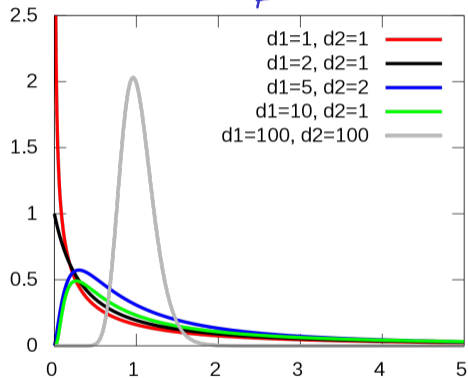
- ▶ The F-statistic follows an F-distribution of parameters $d_1 = I - 1$, $d_2 = \sum_{i=1}^I n_i - I = N - I$.
- ▶ The **p-value** of the F-Test is the probability of the random variable F assuming the null hypothesis is greater to the F_{obs} .



F-distribution

$$X \sim F(d_1, d_2) \quad f(x; d_1, d_2) = \frac{1}{x B\left(\frac{d_1}{2}, \frac{d_2}{2}\right)} \sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}$$

F



ANOVA : Example

$$\mu_1 = \mu_2 = \mu_3 = 1.7$$

Group1 = [85, 88, 97, 97, 77, 99, 98, 88, 85, 78, 86, 88, 82, 77, 82, 77, 86]

Group2 = [91, 92, 83, 85, 87, 84, 82, 88, 75, 76, 92, 88, 89, 83, 93, 94, 80]

Group3 = [79, 78, 88, 94, 92, 85, 93, 85, 82, 81, 80, 75, 84, 97, 88, 87, 77]

Mean Values

GR1	GR2	GR3
86.47	86.0	85.0
$\bar{Y}_{1,17}$	$\bar{Y}_{2,17}$	$\bar{Y}_{3,17}$

$$H_0: \mu_1 = \mu_2 = \mu_3 \quad \text{vs} \quad H_a: \mu_1 \neq \mu_2 \quad \text{or} \quad \mu_1 \neq \mu_3 \quad \text{or} \quad \mu_2 \neq \mu_3$$