$$\overset{t=1}{Y_t = \{4.6}, \overset{2}{6}, \overset{3}{7.2}, \overset{4}{7.3}, \overset{5}{7.9}, \overset{6}{7.8}, \overset{7}{7.9}, \overset{8}{8.1}\}$$

$y_1 \quad y_2 \quad \dots \quad y_8$

logistic.

Διάλεξη 9ης εβδομάδας

$$Y_t = f(t, \beta_1, \beta_2, \beta_3) = \frac{\beta_3}{1 + \beta_2 \exp\{-\beta_1 t\}}$$

$$\frac{1}{f(t)} = \alpha + b \frac{1}{f(t-1)} \qquad t \geq 2$$

Διόρθωση

New Dataset: $\left\{ \left(\overset{x_1}{\frac{1}{y_1}}, \overset{Y_2}{\frac{1}{y_2}}\right), \left(\overset{x_2}{\frac{1}{y_2}}, \overset{Y_2}{\frac{1}{y_3}}\right), \dots \left(\overset{x_7}{\frac{1}{y_7}}, \overset{Y_7}{\frac{1}{y_8}}\right) \right\}$ 7 σημεία.

$$b = \frac{SS_{xy}}{SS_{xx}} \qquad \alpha = \bar{Y} - b\bar{X} = \frac{\sum Y_m}{N} - b \frac{\sum X_m}{N}$$

$$SS_{xy} = \sum X_m Y_m - \frac{\sum X_m \sum Y_m}{N} \qquad SS_{xx} = \sum X_m^2 - \frac{\left(\sum X_m\right)^2}{N} \qquad N = 7$$

X_m	Y_m	X_m^2	$X_m Y_m$
$1/4.6$	$1/6$	$(1/4.6)^2$	$1/(4.6 \cdot 6)$
$1/6$	$1/7.2$	$(1/6)^2$	$1/(6 \cdot 7.2)$
$1/7.2$	$1/7.3$	$(1/7.2)^2$	$\vdots$
$\vdots$	$\vdots$		

$\hat{\beta}_1 = -\ln b \qquad \hat{\beta}_3 = \dfrac{1-b}{a} \qquad \hat{\beta}_2 = \left(\beta_3 / y_1 - 1\right)\exp\{\hat{\beta}_1\}.$

4.6

Άσκηση 2 $\quad S_t = \{S_1, S_2, S_3, S_1, S_2, S_3, \ldots\ldots\}$ $\qquad S_t$

$Y_t = \{85, 105, 130, 115, 140, 158, 143, 170, 197, 188, 206, 227\}$ $\qquad \underline{p=3}$

$Y_t - S_t \approx ?$

$Y_t' = \{\sqcup, 107, 117, 128, 138, 147, 157, 170, 185, 197, 207, \sqcup\}$

$D_t = Y_t - Y_t' = \{\overset{S_1}{\sqcup}, \overset{S_2}{-2}, \overset{S_3}{13}, \overset{S_1}{-13}, \overset{S_2}{2}, \overset{S_3}{11}, \overset{S_1}{-14}, \overset{S_2}{0}, \overset{S_3}{12}, \overset{S_1}{-9}, \overset{S_2}{-1}, \overset{S_3}{\sqcup}\}$ $\qquad t=2,\ldots,11$

$D_t \approx \cancel{T_t} + S_t + R_t - \cancel{T_t} \approx S_t + R_t$

$\bar{D}_t$

$t=1,2,3$

$1,\ldots,p$

$\bar{D}_1 = \dfrac{-13-14-9}{3} = -12 \approx S_1 \qquad \bar{D}_2 = \dfrac{-2+2+0-1}{4} = -1/4 \approx S_2$

$\bar{D}_3 = \dfrac{13+11+12}{3} = 12 \approx S_3$

$$S_1 + S_2 + S_3 = 0 \qquad \sum_{t=1}^{p} S_t = 0$$

$$\hat{S}_1 = \bar{D}_1 - \frac{\bar{D}_1 + \bar{D}_2 + \bar{D}_3}{3} \qquad -\frac{1/4}{3} = -\frac{1}{12}$$

$$\hat{S}_2 = \bar{D}_2 - \frac{\bar{D}_1 + \bar{D}_2 + \bar{D}_3}{3}$$

$$\hat{S}_3 = \bar{D}_3 - \frac{\bar{D}_1 + \bar{D}_2 + \bar{D}_3}{3}$$

$$\hat{S}_1 = -12 + \underbrace{\frac{1}{12}}_{0.1} = -11.9$$

$$\hat{S}_2 = -0.25 + \frac{1}{12} = -0.2$$

$$\hat{S}_3 = 12.1$$

$$\hat{S}_t = [\underset{t=1}{-11.9}, -0.2, 12.1, -11.9, -0.2, 12.1, \ldots\ldots, -11.9, -0.2, \underset{t=12}{12.1}]$$

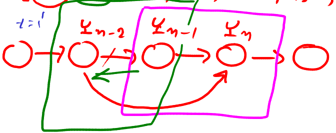
$$Y_t - \hat{S}_t = \ldots \approx T_t + R_t$$

$$Y_t = T_t + S_t + R_t$$

$Y_t = \{85, 105, 130, 115, 140, 158, 143, 170, 197, 188, 206, 227\}$ $\quad y_{n-2} \quad y_n$

$Sum(Y_t) = 1864$

$PACF(2) = ?$



$Y_{n-2} \quad Y_{n-1}$

$Sum(Y * * 2)$

$N = 11$

$r(1) \quad Y_{t-1} \rightarrow Y_t \quad \{(85, 105), (105, 130), (130, 115), \ldots\ldots, (206, 227)\}$

$\quad\quad x \quad\quad y$

$Sum(Y_n * Y_{n-1})$

$$Y_n = A^{(1)} + B^{(1)} Y_{n-1} + \varepsilon_n^{(1)} \longrightarrow \hat{y}_n = \alpha^{(1)} + b^{(1)} y_{n-1} + e_n^{(1)}$$

$$b^{(1)} = \frac{SS_{Y_{n-1}, Y_n}}{SS_{Y_{n-1}, Y_{n-1}}} \qquad \alpha^{(1)} = \bar{Y}_n - b^{(1)} \bar{Y}_{n-1}$$

$\sum y_n = 1864 - 85 = 1779$

$\sum Y_{n-1} = 1864 - 227 = 1637$

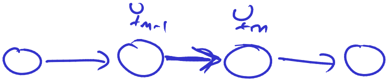
$\sum Y_{n-1} Y_n = 278665$

$\sum y_{n-1}^2 = 310406 - 227^2 = 258877$

$\sum Y_n^2 = 310406 - 85^2 = 303181$

$$SS_{Y_{n-1}, Y_n} = 278665 - \frac{1779 \cdot 1637}{11} = 13917$$

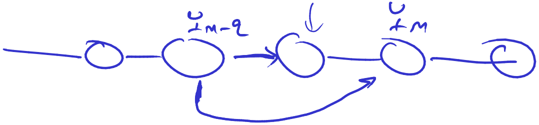
$PACF(1) = ACF(1)$



Dataset $\{(\overset{x}{85}, \overset{y}{105}) \ldots \}$

$$PACF(1) = r = \frac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}} = ACF(1)$$

$ACF(2)$



$\{(\overset{x}{85}, \overset{y}{130}), (105, 115), \overset{10}{\ldots}, (188, 227)\}$

$$ACF(2) = \frac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}}$$

$$SS_{Y_{n-1}Y_{n-1}} = 258877 - \frac{1637^2}{11} = 15261$$

$$b^{(1)} = \frac{13917}{15261} = 0.91 \qquad \alpha^{(1)} = \frac{1779}{11} - 0.91 \cdot \frac{1637}{11} = 26.$$

$$\hat{y}_n = 26 + 0.91\, y_{n-1}$$

#2 $\quad Y_{n-1} \to Y_{n-2}$ $\qquad \hat{y}_{n-2} = \alpha^{(2)} + b^{(2)} y_{n-1}$

$$\begin{array}{l} \quad\; y_{n-1} \;\; y_{n-2} \\ \{(105, 85), (130, 105), \dots, (227, 206)\} \end{array}$$

$\sum Y_{n-1} = 1779$ (ίδιες τιμές με τα αθροίσματα των y_n του προηγούμενου προβλήματος)

$\sum Y^2_{n-1} = 303181 \qquad \sum Y_{n-2} Y_{n-1} = 278665$

$\sum Y_{n-2} = 1637 \quad , \quad \sum Y^2_{n-2} = 258877$

$$SS_{Y_{n-1}Y_{n-2}} = 13917$$

$$SS_{Y_{n-1}Y_{n-1}} = 303181 - \frac{1779^2}{11} = 15468$$

$$b^{(2)} = \frac{13917}{15468} = 0.9 \qquad \alpha^{(2)} = \overline{Y}_{n-2} - b^{(2)}\,\overline{Y}_{n-1} = 6$$

$$\hat{y}_{n-2} = 6 + 0.9 \cdot y_{n-1}$$

Σφάλματα $e_n^{(1)}$ και $e_n^{(2)}$

$$e_n^{(1)} = y_n - \hat{y}_n = \begin{bmatrix} 105 - (26 + 0.91 \cdot 85) \\ 130 - (26 + 0.91 \cdot 105) \\ \vdots \\ 227 - (26 + 0.91 \cdot 206) \end{bmatrix}$$

y_1

Y_n

y_{10}

$$e^{(2)}_{n-2} = y_{n-2} - \hat{y}_{n-2} = \begin{bmatrix} 85 - (6 + 0.9 \cdot 105) \\ \vdots \\ 206 - (6 + 0.9 \cdot 227) \end{bmatrix}$$

$n = 3$

x_1

x_n

x_{10}

$t = 2 \quad n - 2 = 2$

$n = 4$

$n \geq 3$

$x_1 = e^{(2)}_{4-2}$

$\{(x_1, y_1), \dots, (x_{10}, y_{10})\}$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

$12 - 4 = 8$

$$\hat{y}_n = \alpha + b^{(1)} y_{n-1} + b^{(2)} y_{n-2} + b^{(3)} y_{n-3}$$

$$\{(85, 105, 130, 115), (\qquad), (\qquad)\}$$

$$X = \begin{bmatrix} 1 & 85 & 105 & 130 \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \\ 1 & & & \end{bmatrix} \qquad Y = \begin{bmatrix} 115 \\ \vdots \\ \vdots \end{bmatrix} \qquad \hat{P} = \begin{bmatrix} \hat{\alpha} \\ \hat{b}^{(1)} \\ \hat{b}^{(2)} \\ \hat{b}^{(3)} \end{bmatrix}$$

$$\hat{P} = (X^T X)^{-1} X^T Y$$